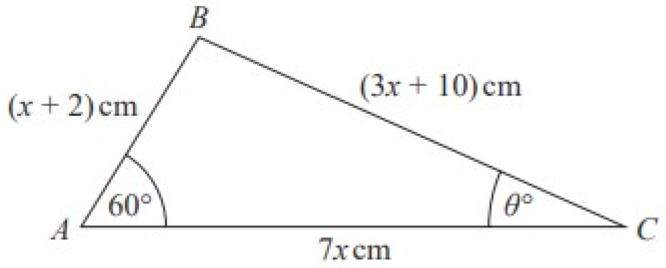


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

(a) Show that the equation

$$4\tan x = 5\cos x$$

can be written as

$$5\sin^2 x + 4\sin x - 5 = 0$$

(3)

(b) Hence solve, for $0 < x \leqslant 360°$

$$4\tan x = 5\cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4\tan 3x = 5\cos 3x$$

in the interval $0 < x \leqslant 1800°$, explaining briefly the reason for your answer.

(2)

(a) Sketch the graph of $y = \sin(x - 30)$ for x in the interval $0 \leq x < 360°$ **(2)**

(b) Find all values for x in the interval $0 \leq x < 360°$, for which

$$\sin(x - 30) = 0.3$$

(4)

Give your answers to one decimal place.

a) (i) To get an equation, we can use cosine rule since we have one angle with all 3 sides.

$$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \angle BAC$$

$$(3x+10)^2 = (x+2)^2 + (7x)^2 - \cancel{2}(x+2)(7x) \cancel{\cos 60°} \quad (1)$$

$$9x^2 + 60x + 100 = x^2 + 4x + 4 + 49x^2 - 7x^2 - 14x \quad (1)$$

$$9x^2 + 60x + 100 = 43x^2 - 10x + 4$$

$$34x^2 - 70x - 96 = 0$$

$$\therefore 17x^2 - 35x - 48 = 0 \quad (1)$$

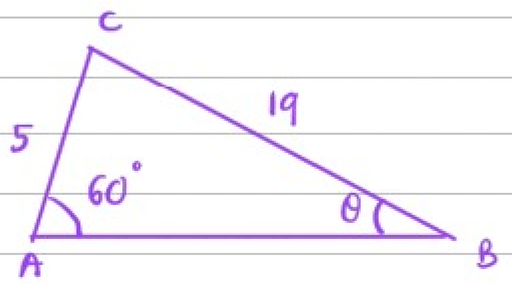
(ii) $17x^2 - 35x - 48 = (17x + 16)(x - 3) = 0$

$$x = \frac{-16}{17}, \quad x = 3 \quad (1)$$

length cannot be negative value

$\therefore$ since x can only be positive, $x = 3$ is the only solution.

b) when $x = 3$,



$AB = (x+2)\,cm = 5\ cm$

$BC = (3x+10)\,cm = 19\ cm$

using sin rule to get the angle θ :

$$\frac{\sin\theta}{5\ cm} = \frac{\sin 60°}{19\ cm} \quad (1)$$

$$\sin\theta = \frac{5}{19}\sin 60°$$

$$\theta = \sin^{-1}\frac{5\sqrt{3}}{38} \quad (1)$$

$$= 13.17° \approx 13.2° \ (1\ d.p.)$$

(1)

a) $4 \tan x = 5 \cos x$

$$4 \frac{\sin x}{\cos x} = 5 \cos x \quad 1$$

$$4 \sin x = 5 \cos^2 x$$

$$4 \sin x = 5(1 - \sin^2 x) \quad 1$$

$$4 \sin x = 5 - 5 \sin^2 x$$

$$5 \sin^2 x + 4 \sin x - 5 = 0 \quad 1$$

b) $4 \tan x = 5 \cos x \equiv 5 \sin^2 x + 4 \sin x - 5 = 0$ 1

Let $a = \sin x$, $5a^2 + 4a - 5 = 0$

$$a = \frac{-2 \pm \sqrt{29}}{5}$$

$$\sin x = \frac{-2 \pm \sqrt{29}}{5} \quad 1$$

$$\boxed{\sin x = \frac{-2 + \sqrt{29}}{5}}, \quad \frac{-2 - \sqrt{29}}{5}$$

not a solution because the value is less than -1, so it is outside of range of $\sin x$

Sin	ALL
2nd	1st
3rd	4th
tan	cos

$$x = \sin^{-1} \frac{-2 + \sqrt{29}}{5} \quad 1$$

take the value from 2nd quadrant also since sin is +ve on this quadrant.

$= 42.6°$ and $180° - 42.6°$

1 $= 137.4°$

c) for interval of $0^\circ < x \leq 360^\circ$,

The number of solution would be $2 \times 3 = 6$ because the x value decrease by factor of 3. (1)

for interval of $0^\circ < x \leq 1800^\circ$, which is 5 times larger than the previous interval,

the number of solutions would be $6 \times 5 = 30$. (1)